

is selected, as can be visualized in the Figs. 13 and 14. This choice of the nominal transmission function is more or less dictated by heuristic principles and can possibly be improved by an optimized criterion. Since the nominal transmission function only has a value other than zero within the fade-out range, it does not interfere with other frequency ranges.

Another restriction of the class of potential compensation pulses that has to be considered though is the criterion of orthogonality in order to keep the reception of the various filters free from any interference. In order for the data in the receiver to be capable of being demodulated and separated with the help of a DFT-Transform, it is necessary that the compensation pulse is orthogonal to the transmission functions of all the subchannels used.

The indices of all the charged subcarriers may be combined in the quantity  $M$ . The  $K$  functions

$$h_k(n) = \frac{1}{\sqrt{M}} e^{j \frac{2\pi}{M} kn} \quad \text{für } k \in \mathcal{K}, n = 0, 1, \dots, M-1 \quad (16)$$

form an orthonormal base for the  $K$  dimensional subspace  $K$ . The indices of all the subcarriers be combined in the quantity  $M$ ,  $M = 0, 1, \dots, M-1$ . The functions

$$h_k(n) = \frac{1}{\sqrt{M}} e^{j \frac{2\pi}{M} kn} \quad \text{für } k, n = 0, 1, \dots, M-1 \quad (17)$$

form the  $M$  dimensional space  $M$ ,  $K$  being a subspace of  $M$ . The transmission functions of the subcarriers used are located in the subspace  $K$ . The compensation pulse must be orthogonal to these functions, i.e., it must lie in a subspace  $L = K^\perp$  which is normal to  $K$ . The space that

presents itself here is the difference  $L = M/K$ . For this  $L = M - K$  dimensional subspace, the functions

$$h_k(n) = \frac{1}{\sqrt{M}} e^{j \frac{2\pi}{M} kn} \quad \text{für } k \in \mathcal{L}, n = 0, 1, \dots, M-1 \quad (18)$$

constitute an orthonormal base. The quantity  $L$  is defined as  $L = M/K$ . The compensation pulse can now be represented by the linear combination of the base vectors (18)

$$g(n) = \sum_{l \in \mathcal{L}} c_l h_l(n) \quad \text{bzw.} \quad g = Hc \quad (19)$$

in the manner of writing vectors, whereas  $g = (g(0)g(1)\dots g(M-1))^T$ . In the column vector  $c$ , the coefficients  $c_l$  of the linear combination are combined. The columns of the matrix  $H$  are the base vectors (18).

$$H = [h_{l_0} h_{l_1} \dots h_{l_{L-1}}] \quad \text{mit} \quad \{l_0 l_1 \dots l_{L-1}\} = \mathcal{L} . \quad (20)$$

To calculate the compensation pulse, the following optimization calculation can be written down:

$$g(n) = \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} |G(e^{j\theta}) - S(e^{j\theta})|^2 d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} |G(e^{j\theta})|^2 d\theta \quad (21)$$

$G(e^{j\theta})$  is the Fourier Transform of  $g(n)$  which is the compensation pulse looked for. Accordingly, the minimization occurs through all the functions of the space  $\mathcal{L}$  which is normal to the transmission functions used. The first integral constitutes the deviation of  $G(e^{j\theta})$  from the nominal transmission function  $S(e^{j\theta})$ . This deviation is calculated within the subband

As already mentioned, the nominal transmission function  $S(e^{j\theta})$  equals zero outside this band. The second integral calculates the energy of  $G(e^{j\theta})$  within the bands  $\theta_{l1} \leq \theta < \theta_{l2}$ . As already mentioned, the compensation pulse is to possess a highly attenuated transmission function outside its band. Summation occurs over ranges in which suppression is desired to be of different strength. Attenuation will need to be higher within the fade-out range than outside the same. This behavior can be adjusted by means of the weighting coefficients  $W_l$ .

Multiplying out (21) yields

$$g(n) = \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} \left( G(e^{j\theta}) - S(e^{j\theta}) \right)^* \left( G(e^{j\theta}) - S(e^{j\theta}) \right)^T d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} G(e^{j\theta})^* G(e^{j\theta})^T d\theta \quad (22)$$

$$= \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} \left( g^t \psi^*(e^{j\theta}) - s^t \psi^*(e^{j\theta}) \right) \left( \psi^T(e^{j\theta}) g - \psi^T(e^{j\theta}) s \right) d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} g^t \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g d\theta \quad (23)$$

$$= \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} \left( g^t \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g - s^t \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g - g^t \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) s + s^t \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) s \right) d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} g^t \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g d\theta \quad (24)$$